UNSTEADY TEMPERATURE DISCONTINUITY

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The condition of an unsteady temperature discontinuity is derived for a one-dimensional heat flux in a gas at rest.

The condition of temperature discontinuity first obtained by Smoluchowski and subsequently refined by a number of other authors [1] is valid only in the steady-state case or for slowly varying flows. If the characteristic time of the process is commensurable with the average time between collisions of molecules, the condition of the temperature discontinuity must change.

We will consider a one-dimensional heat flux in a gas at rest. From the equations of continuity and motion follow respectively [2]

$$\partial\rho/\partial t = 0, \tag{1}$$

$$\partial P_{11}/\partial x = 0, \tag{2}$$

where the x axis is directed along the normal to the surface.

To derive the condition of an unsteady temperature discontinuity, we will use the equation and boundary condition in a 13-moment approximation [2] which on the basis of (2) and the ratio $p_{11}/p \ll 1$ are written:

$$\frac{\partial S}{\partial t} + 5Rp\frac{\partial T}{\partial x} - 2RT\frac{\partial p}{\partial x} + \frac{5}{2\lambda}R^2T\rho S = 0, \tag{3}$$

$$\left(\frac{2\pi}{RT}\right)^{1/2}\frac{S}{p} + \frac{8\sigma}{2-\sigma}\left(1-\frac{T_w}{T}\right) = 0. \tag{4}$$

We differentiate (4) with respect to time, considering the wall temperature T_w to be constant and taking into account (1),

$$\frac{\partial S}{\partial t} = -\frac{8\sigma}{2-\sigma}\rho\left(\frac{R^3T_w}{2\pi}\right)^{1/2}\frac{\partial T}{\partial t}. \tag{5}$$

On the basis of (3)-(5) we calculate the temperature discontinuity

$$T(0) - T_w = -\frac{2\lambda}{5R^2\rho T_w}\cdot\frac{\partial T}{\partial t} + \frac{(2-\sigma)\lambda}{8\sigma\rho}\left(\frac{8\pi}{R^3T_w}\right)^{1/2}\frac{\partial T}{\partial x} - \frac{(2-\sigma)\lambda}{40\sigma\rho^2}\left(\frac{\pi}{T_w}\right)^{1/2}\left(\frac{2}{R}\right)^{5/2}\frac{\partial p}{\partial x}. \tag{6}$$

Finally, using relationship $\lambda = (15/4)R\rho l(2RT_w/\pi)^{1/2}$, we find from (6)

$$T(0) - T_w = \frac{15}{8}\cdot\frac{2-\sigma}{\sigma}l\frac{\partial T}{\partial x} - \frac{3l}{(2\pi RT_w)^{1/2}}\cdot\frac{\partial T}{\partial t} - \frac{2-\sigma}{\sigma}\cdot\frac{3l}{4R\rho}\cdot\frac{\partial p}{\partial x}. \tag{7}$$

If the heat flux and viscous stress tensor, which in the 13-moment approximation are independent variables, do not depend on the coordinate, then from the 13-moment approximation we can obtain easily a hyperbolic equation of heat conductivity [2]. In this same case, with consideration of (2), $\partial p/\partial x = 0$, i.e., condition (7) without the last term is the correct boundary condition for the hyperbolic equation of heat transfer in a rarefied medium.

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It should be noted that an empirical boundary condition was used in [3] for the problem of a thermal boundary layer

$$T(0) - T_w = \varkappa \frac{\partial T}{\partial x} - \vartheta \frac{\partial T}{\partial t}, \qquad (8)$$

where the ratio $\varkappa/\vartheta$ was taken as equal to the average thermal velocity of the molecules. In this case the authors note that not only the boundary conditions but also the equations used ought to be modified.

In the Navier–Stokes approximation, when $S = -2\lambda(\partial T/\partial x)$, relationship (7) changes to the known condition of a steady-state temperature discontinuity.

NOTATION

ρ, λ	are the density and heat conductivity of the gas;
p	is the pressure;
P_{11}, p_{11}	are the stress tensor and viscous stress tensor;
S	is the doubled heat flux;
l	is the mean free path;
σ	is the accomodation coefficient.

LITERATURE CITED

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3. W. Fiszdon and J. Luboński, Bull. de l'Acad. Polonaise des Sciences, Sér. des Sciences Techniques, 17, No. 2 (1969).